



Supplement of

Decay-phase aerosol dynamics of an indoor particle source have a significant role in exposure analysis

Kuisma Vesisenaho et al.

Correspondence to: Kuisma Vesisenaho (kuisma.vesisenaho@tuni.fi)

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S1 Solutions of the differential equations

S1.1 Decay function considering ventilation and deposition

The differential equation describing time behaviour of indoor LDSA concentration, when ventilation and deposition are considered, can be modified to the form of

$$\begin{aligned}\frac{dC_{\text{LDSA},i}}{dt} &= -D_{\text{LDSA}} C_{\text{LDSA},i} + S_{\text{LDSA}} \\ \Leftrightarrow dC_{\text{LDSA},i} + (D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}) dt &= 0\end{aligned}$$

From this, can be read that

$$\begin{aligned}M &= D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}, \\ N &= 1.\end{aligned}$$

Then

$$\frac{\partial M}{\partial C_{\text{LDSA},i}} = D_{\text{LDSA}} \neq 0 = \frac{\partial N}{\partial t},$$

which means that the differential equation is not exact and an integrating factor has to be found. Fortunately, it is noticed that

$$\begin{aligned}f(C_{\text{LDSA},i}) &= -\frac{1}{M} \left(\frac{\partial M}{\partial C_{\text{LDSA},i}} - \frac{\partial N}{\partial t} \right) \\ &= -\frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}} (D_{\text{LDSA}} - 0) \\ &= -\frac{D_{\text{LDSA}}}{D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}}\end{aligned}$$

depends only on $C_{\text{LDSA},i}$, not on t . Thus, the integrating factor is

$$\mu(C_{\text{LDSA},i}) = e^{\int f(C_{\text{LDSA},i}) dC_{\text{LDSA},i}} = \frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}}.$$

Multiplying both sides of the differential equation with the integrating factor, gives a separable differential equation

$$\begin{aligned}\frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}} dC_{\text{LDSA},i} + \frac{D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}}{D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}} dt &= 0 \\ \Leftrightarrow \frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}} dC_{\text{LDSA},i} + dt &= 0 \\ \Leftrightarrow \frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}} dC_{\text{LDSA},i} &= -dt.\end{aligned}$$

This equation can be solved by integrating over the time interval $[0, t]$, when the interval of indoor LDSA concentration is $[C_{\text{LDSA},i,0}, C_{\text{LDSA},i}]$, where $C_{\text{LDSA},i,0}$ is the initial indoor LDSA concentration. The integration gives

$$\begin{aligned}
\int_{C_{\text{LDSA},i,0}}^{C_{\text{LDSA},i}} \frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}} dC_{\text{LDSA},i} &= \int_0^t -dt \\
\Leftrightarrow \int_{C_{\text{LDSA},i,0}}^{C_{\text{LDSA},i}} \frac{\ln(D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}})}{D_{\text{LDSA}}} &= \int_0^t -dt \\
\Leftrightarrow \frac{\ln(D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}})}{D_{\text{LDSA}}} - \frac{\ln(D_{\text{LDSA}} C_{\text{LDSA},i,0} - S_{\text{LDSA}})}{D_{\text{LDSA}}} &= -t \\
\Leftrightarrow \frac{\ln(D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}})}{D_{\text{LDSA}}} &= -t + \frac{\ln(D_{\text{LDSA}} C_{\text{LDSA},i,0} - S_{\text{LDSA}})}{D_{\text{LDSA}}} \\
\Leftrightarrow \ln(D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}}) &= -D_{\text{LDSA}} t + \ln(D_{\text{LDSA}} C_{\text{LDSA},i,0} - S_{\text{LDSA}}) \\
\Leftrightarrow D_{\text{LDSA}} C_{\text{LDSA},i} - S_{\text{LDSA}} &= e^{-D_{\text{LDSA}} t} (D_{\text{LDSA}} C_{\text{LDSA},i,0} - S_{\text{LDSA}}) \\
\Leftrightarrow D_{\text{LDSA}} C_{\text{LDSA},i} &= e^{-D_{\text{LDSA}} t} (D_{\text{LDSA}} C_{\text{LDSA},i,0} - S_{\text{LDSA}}) + S_{\text{LDSA}} \\
\Leftrightarrow C_{\text{LDSA},i} &= e^{-D_{\text{LDSA}} t} \left(C_{\text{LDSA},i,0} - \frac{S_{\text{LDSA}}}{D_{\text{LDSA}}} \right) + \frac{S_{\text{LDSA}}}{D_{\text{LDSA}}}
\end{aligned}$$

as a solution for the differential equation considering ventilation and deposition. As time approaches infinity, gradient of LDSA concentration approaches zero and LDSA concentration approaches background concentration $C_{\text{LDSA},i,\text{bg}}$ resulted from background source. Consequently, from the original differential equation, the background source S_{LDSA} can be formulated as

$$\begin{aligned}
0 &= -D_{\text{LDSA}} C_{\text{LDSA},i} + S_{\text{LDSA}} \\
\Leftrightarrow S_{\text{LDSA}} &= D_{\text{LDSA}} C_{\text{LDSA},i}.
\end{aligned}$$

It has to be noted that this step includes the assumption of a constant dilution coagulation coefficient for LDSA. Using the equation of background source, the auxiliary variable b can be expressed as

$$\begin{aligned}
C_{\text{LDSA},i} &= e^{-D_{\text{LDSA}} t} \left(C_{\text{LDSA},i,0} - \frac{D_{\text{LDSA}} C_{\text{LDSA},i}}{D_{\text{LDSA}}} \right) + \frac{D_{\text{LDSA}} C_{\text{LDSA},i}}{D_{\text{LDSA}}} \\
&= C_{\text{LDSA},i,\text{bg}} + (C_{\text{LDSA},i,0} - C_{\text{LDSA},i,\text{bg}}) e^{-D_{\text{LDSA}} t}.
\end{aligned}$$

S1.2 Decay function considering ventilation, deposition, and coagulation

The differential equation describing time behaviour of indoor LDSA concentration, when ventilation, deposition, and coagulation are considered, can be modified to the form of

$$\begin{aligned}
\frac{dC_{\text{LDSA},i}}{dt} &= -D_{\text{LDSA}} C_{\text{LDSA},i} - K_{\text{LDSA}} C_{\text{LDSA},i}^2 + S_{\text{LDSA}} \\
\Leftrightarrow dC_{\text{LDSA},i} + (D_{\text{LDSA}} C_{\text{LDSA},i} + K_{\text{LDSA}} C_{\text{LDSA},i}^2 - S_{\text{LDSA}}) dt &= 0
\end{aligned}$$

From this, can be read that

$$M = D_{\text{LDSA}} C_{\text{LDSA},i} + K_{\text{LDSA}} C_{\text{LDSA},i}^2 - S_{\text{LDSA}},$$

$$N = 1.$$

Then

$$\frac{\partial M}{\partial C_{\text{LDSA},i}} = D_{\text{LDSA}} + 2K_{\text{LDSA}} C_{\text{LDSA},i} \neq 0 = \frac{\partial N}{\partial t},$$

which means that the differential equation is not exact and an integrating factor has to be found. Fortunately, it is noticed that

$$\begin{aligned} f(C_{\text{LDSA},i}) &= -\frac{1}{M} \left(\frac{\partial M}{\partial C_{\text{LDSA},i}} - \frac{\partial N}{\partial t} \right) \\ &= -\frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} + K_{\text{LDSA}} C_{\text{LDSA},i}^2 - S_{\text{LDSA}}} (D_{\text{LDSA}} + 2K_{\text{LDSA}} C_{\text{LDSA},i} - 0) \\ &= -\frac{D_{\text{LDSA}} + 2K_{\text{LDSA}} C_{\text{LDSA},i}}{D_{\text{LDSA}} C_{\text{LDSA},i} + K_{\text{LDSA}} C_{\text{LDSA},i}^2 - S_{\text{LDSA}}} \end{aligned}$$

depends only on $C_{\text{LDSA},i}$, not on t . Thus, the integrating factor is

$$\mu(C_{\text{LDSA},i}) = e^{\int f(C_{\text{LDSA},i}) dC_{\text{LDSA},i}} = \frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} + K_{\text{LDSA}} C_{\text{LDSA},i}^2 - S_{\text{LDSA}}}.$$

Multiplying both sides of the differential equation with the integrating factor, gives a separable differential equation

$$\begin{aligned} &\frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} + K_{\text{LDSA}} C_{\text{LDSA},i}^2 - S_{\text{LDSA}}} dC_{\text{LDSA},i} + \frac{D_{\text{LDSA}} C_{\text{LDSA},i} + K_{\text{LDSA}} C_{\text{LDSA},i}^2 - S_{\text{LDSA}}}{D_{\text{LDSA}} C_{\text{LDSA},i} + K_{\text{LDSA}} C_{\text{LDSA},i}^2 - S_{\text{LDSA}}} dt = 0 \\ \Leftrightarrow &\frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} + K_{\text{LDSA}} C_{\text{LDSA},i}^2 - S_{\text{LDSA}}} dC_{\text{LDSA},i} + dt = 0 \\ \Leftrightarrow &\frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} + K_{\text{LDSA}} C_{\text{LDSA},i}^2 - S_{\text{LDSA}}} dC_{\text{LDSA},i} = -dt. \end{aligned}$$

This equation can be solved by integrating over the time interval $[0, t]$, when the interval of indoor LDSA concentration is $[C_{\text{LDSA},i,0}, C_{\text{LDSA},i}]$, where $C_{\text{LDSA},i,0}$ is the initial indoor LDSA concentration. The integration gives

$$\begin{aligned} &\int_{C_{\text{LDSA},i,0}}^{C_{\text{LDSA},i}} \frac{1}{D_{\text{LDSA}} C_{\text{LDSA},i} + K_{\text{LDSA}} C_{\text{LDSA},i}^2 - S_{\text{LDSA}}} dC_{\text{LDSA},i} = \int_0^t -dt \\ \Leftrightarrow &-\frac{2}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}} S_{\text{LDSA}}}} \Big/_{C_{\text{LDSA},i,0}}^{C_{\text{LDSA},i}} \operatorname{atanh} \left(\frac{D_{\text{LDSA}} + 2K_{\text{LDSA}} C_{\text{LDSA},i}}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}} S_{\text{LDSA}}}} \right) = \Big/_0^t -t \\ \Leftrightarrow &-\frac{2}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}} S_{\text{LDSA}}}} \left[\operatorname{atanh} \left(\frac{D_{\text{LDSA}} + 2K_{\text{LDSA}} C_{\text{LDSA},i}}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}} S_{\text{LDSA}}}} \right) - \operatorname{atanh} \left(\frac{D_{\text{LDSA}} + 2K_{\text{LDSA}} C_{\text{LDSA},i,0}}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}} S_{\text{LDSA}}}} \right) \right] = -t \\ \Leftrightarrow &\frac{2}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}} S_{\text{LDSA}}}} \left[\operatorname{atanh} \left(\frac{D_{\text{LDSA}} + 2K_{\text{LDSA}} C_{\text{LDSA},i}}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}} S_{\text{LDSA}}}} \right) - \operatorname{atanh} \left(\frac{D_{\text{LDSA}} + 2K_{\text{LDSA}} C_{\text{LDSA},i,0}}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}} S_{\text{LDSA}}}} \right) \right] = t \end{aligned}$$

Using formula $\text{atanh}(z) = \frac{1}{2} \ln\left(\frac{1+z}{1-z}\right)$ and auxiliary variables $z = \frac{D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i}}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}}$ and $z_0 = \frac{D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,0}}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}}$, variable z can be solved as

$$\begin{aligned}
& \frac{2}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} \left(\frac{1}{2} \ln\left(\frac{1+z}{1-z}\right) - \frac{1}{2} \ln\left(\frac{1+z_0}{1-z_0}\right) \right) = t \\
& \Leftrightarrow \frac{1}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} \left(\ln\left(\frac{1+z}{1-z}\right) - \ln\left(\frac{1+z_0}{1-z_0}\right) \right) = t \\
& \Leftrightarrow \ln\left(\frac{1+z}{1-z}\right) - \ln\left(\frac{1+z_0}{1-z_0}\right) = t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}} \\
& \Leftrightarrow e^{\ln\left(\frac{1+z}{1-z}\right) - \ln\left(\frac{1+z_0}{1-z_0}\right)} = e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} \\
& \Leftrightarrow \frac{1+z}{1-z} \frac{1-z_0}{1+z_0} = e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} \\
& \Leftrightarrow \frac{1+z}{1-z} = \frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} \\
& \Leftrightarrow 1+z = (1-z) \frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} \\
& \Leftrightarrow z \left(1 + \frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} \right) = \frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} - 1 \\
& \Leftrightarrow z = \frac{\frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} - 1}{\frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} + 1}.
\end{aligned}$$

Substituting $z = \frac{D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i}}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}}$, LDSA-concentration can be solved

$$\begin{aligned}
& \frac{D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i}}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} = \frac{\frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} - 1}{\frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} + 1} \\
& \Leftrightarrow D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i} = \frac{\frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} - 1}{\frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} + 1} \sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}} \\
& \Leftrightarrow 2K_{\text{LDSA}}C_{\text{LDSA},i} = \frac{\frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} - 1}{\frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} + 1} \sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}} - D_{\text{LDSA}} \\
& \Leftrightarrow C_{\text{LDSA},i} = \frac{\frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} - 1}{\frac{1+z_0}{1-z_0} e^{t\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} + 1} \frac{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}}{2K_{\text{LDSA}}} - \frac{D_{\text{LDSA}}}{2K_{\text{LDSA}}}.
\end{aligned}$$

Using an auxiliary variable $b = \sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S}$, this can be expressed as

$$\begin{aligned}
C_{\text{LDSA},i} &= \frac{\frac{1+z_0}{1-z_0}e^{bt} - 1}{\frac{1+z_0}{1-z_0}e^{bt} + 1} \frac{b}{2K_{\text{LDSA}}} - \frac{D_{\text{LDSA}}}{2K_{\text{LDSA}}} \\
&= \frac{\frac{1+z_0}{1-z_0}e^{bt} + 1 - 2}{\frac{1+z_0}{1-z_0}e^{bt} + 1} \frac{b}{2K_{\text{LDSA}}} - \frac{D_{\text{LDSA}}}{2K_{\text{LDSA}}} \\
&= \left(1 - \frac{2}{\frac{1+z_0}{1-z_0}e^{bt} + 1}\right) \frac{b}{2K_{\text{LDSA}}} - \frac{D_{\text{LDSA}}}{2K_{\text{LDSA}}} \\
&= \frac{1}{2K_{\text{LDSA}}} \left(b - \frac{2b}{\frac{1+z_0}{1-z_0}e^{bt} + 1} - D_{\text{LDSA}}\right) \\
&= \frac{1}{2K_{\text{LDSA}}} \left(b - \frac{2}{\frac{2-1+z_0}{1-z_0}e^{bt} + 1} - D_{\text{LDSA}}\right) \\
&= \frac{1}{2K_{\text{LDSA}}} \left(b - \frac{2b}{\left(\frac{2}{1-z_0} - 1\right)e^{bt} + 1} - D_{\text{LDSA}}\right).
\end{aligned}$$

Substituting $z_0 = \frac{D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,0}}{\sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}}} = \frac{D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,0}}{b}$, it can be written

$$\begin{aligned}
&= \frac{1}{2K_{\text{LDSA}}} \left(b - \frac{2b}{\left(\frac{2}{1 - \frac{D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,0}}{b}} - 1\right)e^{bt} + 1} - D_{\text{LDSA}}\right) \\
&= \frac{1}{2K_{\text{LDSA}}} \left(b - \frac{2b}{\left(\frac{2b}{b - D_{\text{LDSA}} - 2K_{\text{LDSA}}C_{\text{LDSA},i,0}} - 1\right)e^{bt} + 1} - D_{\text{LDSA}}\right) \\
&= \frac{1}{2K_{\text{LDSA}}} \left(b - D_{\text{LDSA}} - 2b \left[\left(\frac{2b}{b - D_{\text{LDSA}} - 2K_{\text{LDSA}}C_{\text{LDSA},i,0}} - 1\right)e^{bt} + 1\right]^{-1}\right).
\end{aligned}$$

As time approaches infinity, gradient of LDSA concentration approaches zero and LDSA concentration approaches background concentration $C_{\text{LDSA},i,\text{bg}}$ resulted from background source. Consequently, from the original differential equation, the background source S_{LDSA} can be formulated as

$$\begin{aligned}
0 &= -D_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}} - K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}}^2 + S_{\text{LDSA}} \\
\Leftrightarrow S_{\text{LDSA}} &= D_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}} + K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}}^2.
\end{aligned}$$

It has to be noted that this step includes the assumption of constant dilution and coagulation coefficients for LDSA. Using the equation of background source, the auxiliary variable b can be expressed as

$$\begin{aligned}
b &= \sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}}S_{\text{LDSA}}} \\
&= \sqrt{D_{\text{LDSA}}^2 + 4K_{\text{LDSA}} \left(D_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}} + K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}}^2 \right)} \\
&= \sqrt{D_{\text{LDSA}}^2 + 4D_{\text{LDSA}}K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}} + 4K_{\text{LDSA}}^2C_{\text{LDSA},i,\text{bg}}^2} \\
&= \sqrt{(D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}})^2} \\
&= D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}}.
\end{aligned}$$

Finally, using this, LDSA-concentration can be written as

$$\begin{aligned}
C_{\text{LDSA},i} &= \frac{1}{2K_{\text{LDSA}}} \left(b - D_{\text{LDSA}} - 2b \left[\left(\frac{2b}{b - D_{\text{LDSA}} - 2K_{\text{LDSA}}C_{\text{LDSA},i,0}} - 1 \right) e^{bt} + 1 \right]^{-1} \right) \\
&= \frac{1}{2K_{\text{LDSA}}} \left(2K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}} - 2(D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}}) \left[\left(\frac{2(D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}})}{D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}} - D_{\text{LDSA}} - 2K_{\text{LDSA}}C_{\text{LDSA},i,0}} - 1 \right) e^{bt} + 1 \right]^{-1} \right) \\
&= \frac{1}{2K_{\text{LDSA}}} \left(2K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}} - 2(D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}}) \left[\left(\frac{2(D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}})}{2K_{\text{LDSA}}(C_{\text{LDSA},i,\text{bg}} - C_{\text{LDSA},i,0})} - 1 \right) e^{bt} + 1 \right]^{-1} \right) \\
&= C_{\text{LDSA},i,\text{bg}} - \frac{D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}}}{K_{\text{LDSA}}} \left[\left(\frac{D_{\text{LDSA}} + 2K_{\text{LDSA}}C_{\text{LDSA},i,\text{bg}}}{K_{\text{LDSA}}(C_{\text{LDSA},i,\text{bg}} - C_{\text{LDSA},i,0})} - 1 \right) e^{bt} + 1 \right]^{-1} \\
&= C_{\text{LDSA},i,\text{bg}} - \frac{b}{K_{\text{LDSA}}} \left[\left(\frac{b}{K_{\text{LDSA}}(C_{\text{LDSA},i,\text{bg}} - C_{\text{LDSA},i,0})} - 1 \right) e^{bt} + 1 \right]^{-1} \\
&= C_{\text{LDSA},i,\text{bg}} - b \left[\left(\frac{b}{C_{\text{LDSA},i,0} - C_{\text{LDSA},i,\text{bg}}} + K_{\text{LDSA}} \right) e^{bt} - K_{\text{LDSA}} \right]^{-1}.
\end{aligned}$$

S2 Floor plans of the measurement sites

Table S1. Description of the acronyms used in floor plans.

Acronym	Definition
BR	Bedroom
H	Hall
K	Kitchen
L	Living room
S	Sauna
STOR	Storage
TERR	Terrace
UTRM	Utility room
WC	Toilet/bathroom
WIC	Walk-in closet

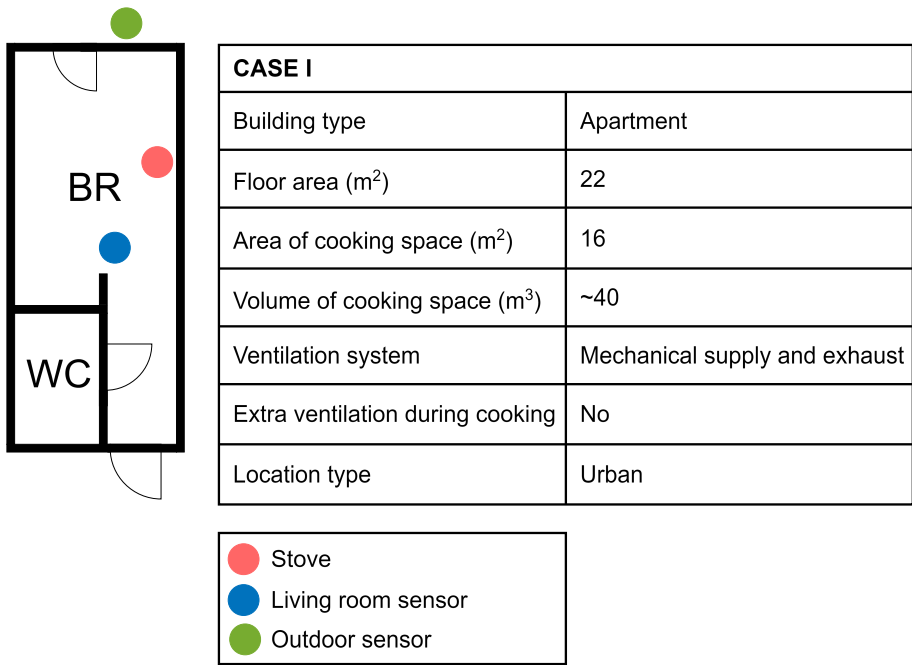


Figure S1. Floor plan and description of the apartment used in Case I.

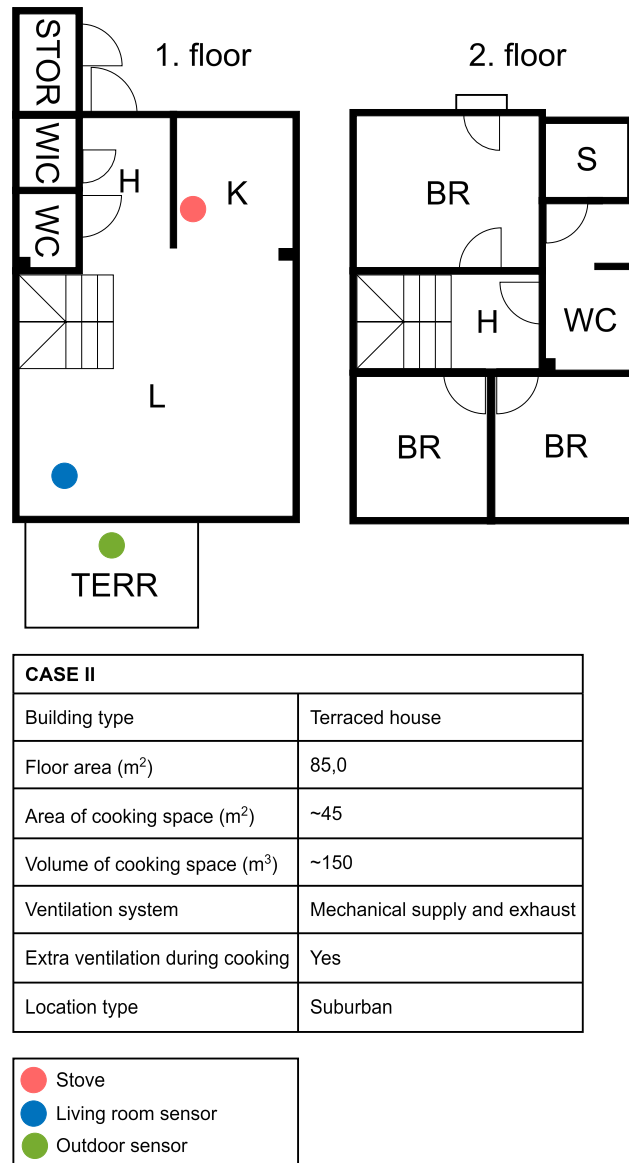


Figure S2. Floor plan and description of the apartment used in Case II

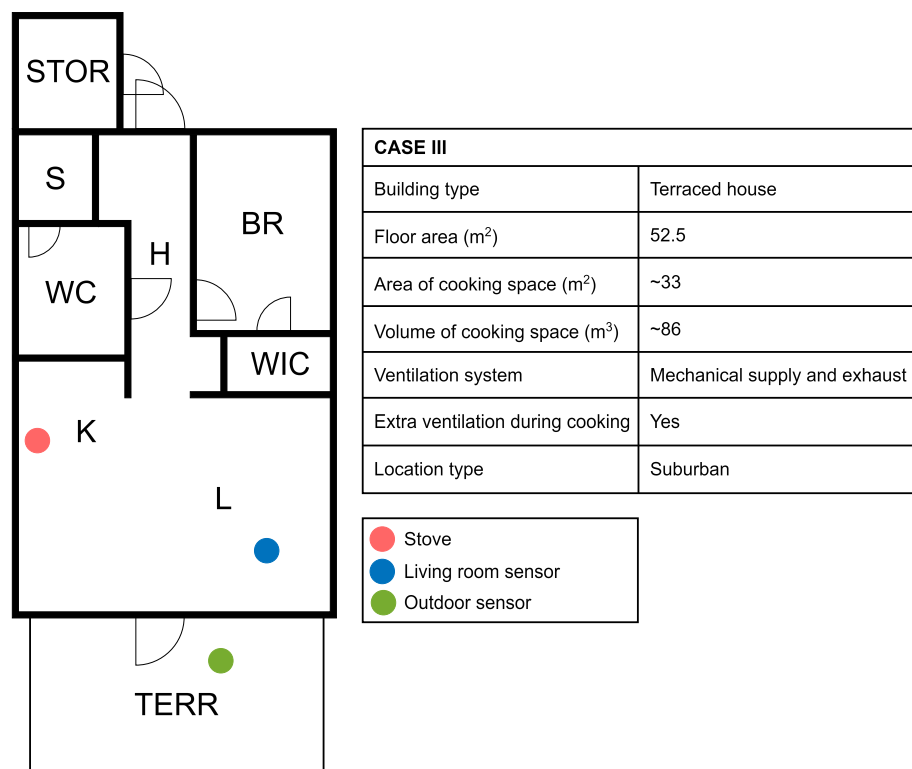


Figure S3. Floor plan and description of the apartment used in Case III

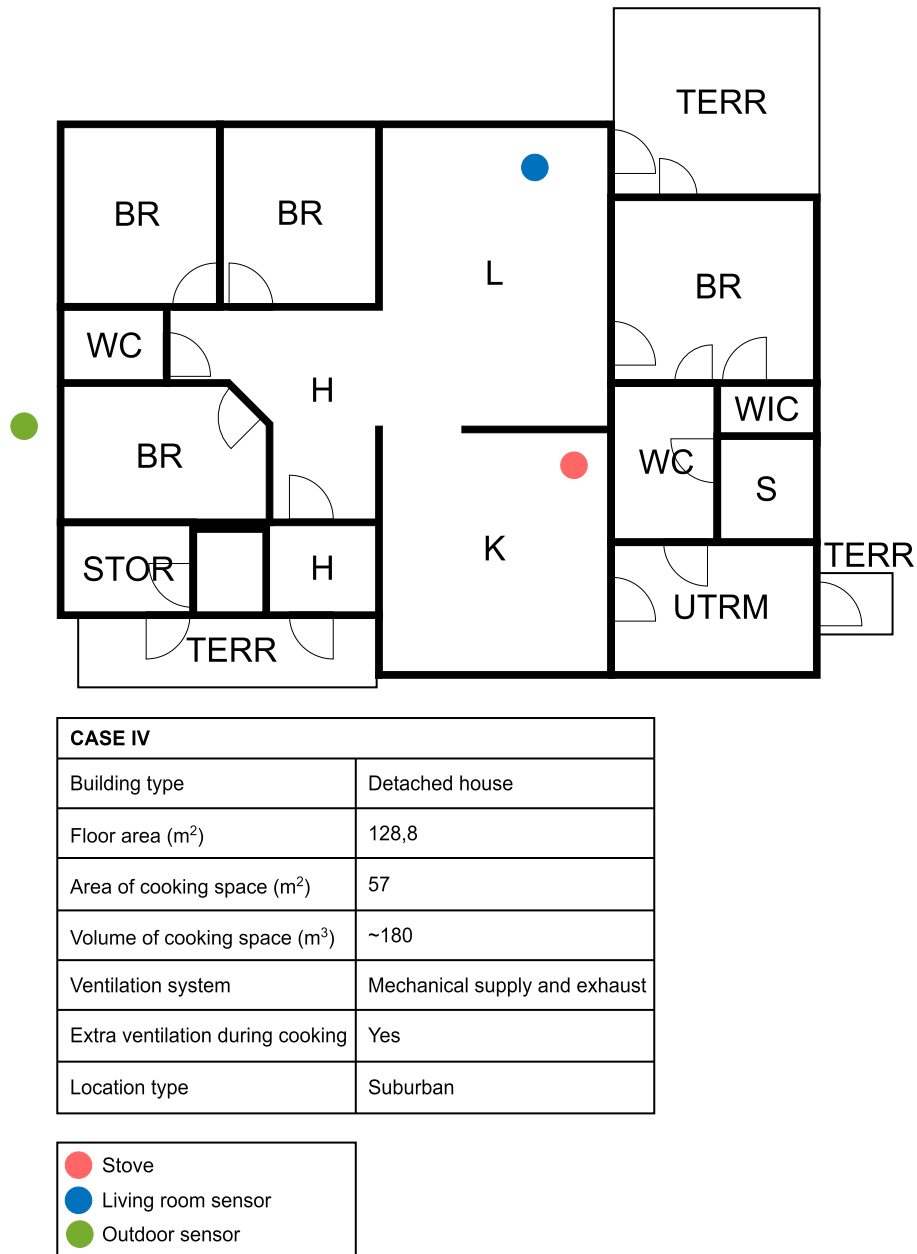


Figure S4. Floor plan and description of the apartment used in Case IV.

S3 Fits of the decay function ignoring coagulation

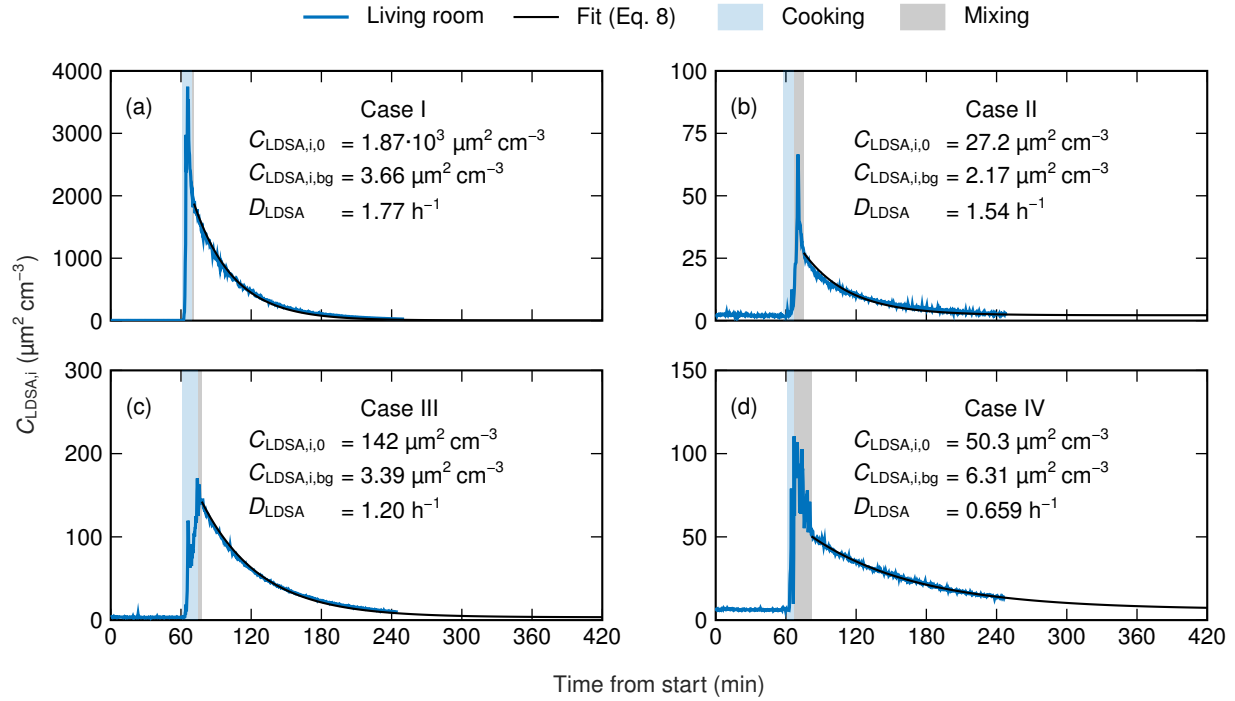


Figure S5. The fits and the fitting parameters of the decay function Eq. (8), not considering coagulation, in Cases I (a), II (b), III (c), and IV (d). Cooking events and mixing phases are highlighted.