S1 Solutions of the differential equations

S1.1 Decay function considering ventilation and deposition

The differential equation describing time behaviour of indoor LDSA concentration, when ventilation and deposition are considered, can be modified to the form of

$$\frac{\mathrm{d}C_{\mathrm{LDSA,i}}}{\mathrm{d}t} = -D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} + S_{\mathrm{LDSA}}$$
$$\Leftrightarrow \mathrm{d}C_{\mathrm{LDSA,i}} + \left(D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}\right)\mathrm{d}t = 0$$

From this, can be read that

$$M = D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}},$$
$$N = 1.$$

Then

$$\frac{\partial M}{\partial C_{\mathrm{LDSA,i}}} = D_{\mathrm{LDSA}} \neq 0 = \frac{\partial N}{\partial t},$$

which means that the differential equation is not exact and an integrating factor has to be found. Fortunately, it is noticed that

$$\begin{aligned} f\left(C_{\mathrm{LDSA,i}}\right) &= -\frac{1}{M}\left(\frac{\partial M}{\partial C_{\mathrm{LDSA,i}}} - \frac{\partial N}{\partial t}\right) \\ &= -\frac{1}{D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}}\left(D_{\mathrm{LDSA}} - 0\right) \\ &= -\frac{D_{\mathrm{LDSA}}}{D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}} \end{aligned}$$

depends only on $C_{\mathrm{LDSA,i}}$, not on t. Thus, the integrating factor is

$$\mu\left(C_{\mathrm{LDSA,i}}\right) = e^{\int f\left(C_{\mathrm{LDSA,i}}\right)\,\mathrm{d}C_{\mathrm{LDSA,i}}} = \frac{1}{D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}}.$$

Multiplying both sides of the differential equation with the integrating factor, gives a separable differential equation

$$\frac{1}{D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}}\mathrm{d}C_{\mathrm{LDSA,i}} + \frac{D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}}{D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}}\mathrm{d}t = 0$$
$$\Leftrightarrow \frac{1}{D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}}\mathrm{d}C_{\mathrm{LDSA,i}} + \mathrm{d}t = 0$$
$$\Leftrightarrow \frac{1}{D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}}\mathrm{d}C_{\mathrm{LDSA,i}} = -\mathrm{d}t.$$

This equation can be solved by integrating over the time interval $[0,t]$, when the interval of indoor LDSA concentration is $[C_{\mathrm{LDSA,i,0}}, C_{\mathrm{LDSA,i}}]$, where $C_{\mathrm{LDSA,i,0}}$ is the initial indoor LDSA concentration. The integration gives

$$\int\limits_{C_{\mathrm{LDSA,i,0}}}^{C_{\mathrm{LDSA,i}}} \frac{1}{D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}} \mathrm{d}C_{\mathrm{LDSA,i}} = \int\limits_{0}^{t} -dt$$

$$\Leftrightarrow \Big/_{C_{\mathrm{LDSA,i,0}}}^{C_{\mathrm{LDSA,i}}} \frac{\ln\left(D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}\right)}{D_{\mathrm{LDSA}}} = \Big/_{0}^{t} -t$$

$$\Leftrightarrow \frac{\ln\left(D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}\right)}{D_{\mathrm{LDSA}}} - \frac{\ln\left(D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}} - S_{\mathrm{LDSA}}\right)}{D_{\mathrm{LDSA}}} = -t$$

$$\Leftrightarrow \frac{\ln\left(D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}\right)}{D_{\mathrm{LDSA}}} = -t + \frac{\ln\left(D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}} - S_{\mathrm{LDSA}}\right)}{D_{\mathrm{LDSA}}}$$

$$\Leftrightarrow \ln\left(D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}}\right) = -D_{\mathrm{LDSA}}t + \ln\left(D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}} - S_{\mathrm{LDSA}}\right)$$

$$\Leftrightarrow D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - S_{\mathrm{LDSA}} = e^{-D_{\mathrm{LDSA}}t}\left(D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}} - S_{\mathrm{LDSA}}\right)$$

$$\Leftrightarrow D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} = e^{-D_{\mathrm{LDSA}}t}\left(D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}} - S_{\mathrm{LDSA}}\right) + S_{\mathrm{LDSA}}$$

$$\Leftrightarrow C_{\mathrm{LDSA,i}} = e^{-D_{\mathrm{LDSA}}t}\left(C_{\mathrm{LDSA,i,0}} - \frac{S_{\mathrm{LDSA}}}{D_{\mathrm{LDSA}}}\right) + \frac{S_{\mathrm{LDSA}}}{D_{\mathrm{LDSA}}}$$

as a solution for the differential equation considering ventilation and deposition. As time approaches infinity, gradient of LDSA concentration approaches zero and LDSA concentration approaches background concentration $C_{\mathrm{LDSA,i,bg}}$ resulted from background source. Consequently, from the original differential equation, the background source S_{LDSA} can be formulated as

$$0 = -D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} + S_{\mathrm{LDSA}}$$

$$\Leftrightarrow S_{\mathrm{LDSA}} = D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}}.$$

It has to be noted that this step includes the assumption of a constant dilution coagulation coefficient for LDSA. Using the equation of background source, the auxiliary variable b can be expressed as

$$C_{\mathrm{LDSA,i}} = e^{-D_{\mathrm{LDSA}}t}\left(C_{\mathrm{LDSA,i,0}} - \frac{D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}}}{D_{\mathrm{LDSA}}}\right) + \frac{D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}}}{D_{\mathrm{LDSA}}}$$

$$= C_{\mathrm{LDSA,i,bg}} + \left(C_{\mathrm{LDSA,i,0}} - C_{\mathrm{LDSA,i,bg}}\right)e^{-D_{\mathrm{LDSA}}t}.$$

S1.2 Decay function considering ventilation, deposition, and coagulation

The differential equation describing time behaviour of indoor LDSA concentration, when ventilation, deposition, and coagulation are considered, can be modified to the form of

$$\frac{\mathrm{d}C_{\mathrm{LDSA,i}}}{\mathrm{d}t} = -D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} - K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}}^2 + S_{\mathrm{LDSA}}$$

$$\Leftrightarrow \mathrm{d}C_{\mathrm{LDSA,i}} + (D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}} + K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}}^2 - S_{\mathrm{LDSA}})\mathrm{d}t = 0$$

From this, can be read that

$$M = D_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} + K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}^2 - S_{\mathrm{LDSA}},$$
$$N = 1.$$

Then

$$\frac{\partial M}{\partial C_{\mathrm{LDSA,i}}} = D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} \neq 0 = \frac{\partial N}{\partial t},$$

which means that the differential equation is not exact and an integrating factor has to be found. Fortunately, it is noticed that

$$\begin{aligned} f(C_{\mathrm{LDSA,i}}) &= -\frac{1}{M}\left(\frac{\partial M}{\partial C_{\mathrm{LDSA,i}}} - \frac{\partial N}{\partial t}\right) \\ &= -\frac{1}{D_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} + K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}^2 - S_{\mathrm{LDSA}}}\left(D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} - 0\right) \\ &= -\frac{D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}}{D_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} + K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}^2 - S_{\mathrm{LDSA}}} \end{aligned}$$

depends only on $C_{\mathrm{LDSA,i}}$, not on t. Thus, the integrating factor is

$$\mu(C_{\mathrm{LDSA,i}}) = e^{\int f(C_{\mathrm{LDSA,i}})\,\mathrm{d}C_{\mathrm{LDSA,i}}} = \frac{1}{D_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} + K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}^2 - S_{\mathrm{LDSA}}}.$$

Multiplying both sides of the differential equation with the integrating factor, gives a separable differential equation

$$\begin{aligned} &\frac{1}{D_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} + K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}^2 - S_{\mathrm{LDSA}}}\mathrm{d}C_{\mathrm{LDSA,i}} + \frac{D_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} + K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}^2 - S_{\mathrm{LDSA}}}{D_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} + K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}^2 - S_{\mathrm{LDSA}}}\mathrm{d}t = 0 \\ \Leftrightarrow\; &\frac{1}{D_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} + K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}^2 - S_{\mathrm{LDSA}}}\mathrm{d}C_{\mathrm{LDSA,i}} + \mathrm{d}t = 0 \\ \Leftrightarrow\; &\frac{1}{D_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} + K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}^2 - S_{\mathrm{LDSA}}}\mathrm{d}C_{\mathrm{LDSA,i}} = -\mathrm{d}t. \end{aligned}$$

This equation can be solved by integrating over the time interval $[0,t]$, when the interval of indoor LDSA concentration is $[C_{\mathrm{LDSA,i,0}}, C_{\mathrm{LDSA,i}}]$, where $C_{\mathrm{LDSA,i,0}}$ is the initial indoor LDSA concentration. The integration gives

$$\begin{aligned} &\int_{C_{\mathrm{LDSA,i,0}}}^{C_{\mathrm{LDSA,i}}} \frac{1}{D_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}} + K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}^2 - S_{\mathrm{LDSA}}}\mathrm{d}C_{\mathrm{LDSA,i}} = \int_0^t -dt \\ \Leftrightarrow\; &-\frac{2}{\sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}} S_{\mathrm{LDSA}}}} \Big/_{C_{\mathrm{LDSA,i,0}}}^{C_{\mathrm{LDSA,i}}} \operatorname{atanh}\left(\frac{D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}}{\sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}} S_{\mathrm{LDSA}}}}\right) = \Big/_0^t - t \\ \Leftrightarrow\; &-\frac{2}{\sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}} S_{\mathrm{LDSA}}}}\left[\operatorname{atanh}\left(\frac{D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}}{\sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}} S_{\mathrm{LDSA}}}}\right) - \operatorname{atanh}\left(\frac{D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i,0}}}{\sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}} S_{\mathrm{LDSA}}}}\right)\right] = -t \\ \Leftrightarrow\; &\frac{2}{\sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}} S_{\mathrm{LDSA}}}}\left[\operatorname{atanh}\left(\frac{D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i}}}{\sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}} S_{\mathrm{LDSA}}}}\right) - \operatorname{atanh}\left(\frac{D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}} C_{\mathrm{LDSA,i,0}}}{\sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}} S_{\mathrm{LDSA}}}}\right)\right] = t \end{aligned}$$

Using formula $\operatorname{atanh}(z) = \frac{1}{2}\ln\left(\frac{1+z}{1-z}\right)$ and auxiliary variables $z = \frac{D_{\mathrm{LDSA}}+2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}}}{\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}$ and $z_0 = \frac{D_{\mathrm{LDSA}}+2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}}}{\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}$, variable z can be solved as

$$
\begin{aligned}
&\frac{2}{\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}\left(\frac{1}{2}\ln\left(\frac{1+z}{1-z}\right)-\frac{1}{2}\ln\left(\frac{1+z_0}{1-z_0}\right)\right)=t\\
&\Leftrightarrow \frac{1}{\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}\left(\ln\left(\frac{1+z}{1-z}\right)-\ln\left(\frac{1+z_0}{1-z_0}\right)\right)=t\\
&\Leftrightarrow \ln\left(\frac{1+z}{1-z}\right)-\ln\left(\frac{1+z_0}{1-z_0}\right)=t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}\\
&\Leftrightarrow e^{\ln\left(\frac{1+z}{1-z}\right)-\ln\left(\frac{1+z_0}{1-z_0}\right)}=e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}\\
&\Leftrightarrow \frac{1+z}{1-z}\frac{1-z_0}{1+z_0}=e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}\\
&\Leftrightarrow \frac{1+z}{1-z}=\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}\\
&\Leftrightarrow 1+z=(1-z)\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}\\
&\Leftrightarrow z\left(1+\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}\right)=\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}-1\\
&\Leftrightarrow z=\frac{\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}-1}{\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}+1}.
\end{aligned}
$$

Substituting $z = \frac{D_{\mathrm{LDSA}}+2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}}}{\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}$, LDSA-concentration can be solved

$$
\begin{aligned}
&\frac{D_{\mathrm{LDSA}}+2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}}}{\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}=\frac{\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}-1}{\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}+1}\\
&\Leftrightarrow D_{\mathrm{LDSA}}+2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}}=\frac{\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}-1}{\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}+1}\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}\\
&\Leftrightarrow 2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i}}=\frac{\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}-1}{\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}+1}\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}-D_{\mathrm{LDSA}}\\
&\Leftrightarrow C_{\mathrm{LDSA,i}}=\frac{\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}-1}{\frac{1+z_0}{1-z_0}e^{t\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}+1}\frac{\sqrt{D_{\mathrm{LDSA}}^2+4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}}{2K_{\mathrm{LDSA}}}-\frac{D_{\mathrm{LDSA}}}{2K_{\mathrm{LDSA}}}.
\end{aligned}
$$

Using an auxiliary variable $b = \sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}}S}$, this can be expressed as

$$C_{\mathrm{LDSA,i}} = \frac{\frac{1+z_0}{1-z_0}e^{bt} - 1}{\frac{1+z_0}{1-z_0}e^{bt} + 1}\frac{b}{2K_{\mathrm{LDSA}}} - \frac{D_{\mathrm{LDSA}}}{2K_{\mathrm{LDSA}}}$$
$$= \frac{\frac{1+z_0}{1-z_0}e^{bt} + 1 - 2}{\frac{1+z_0}{1-z_0}e^{bt} + 1}\frac{b}{2K_{\mathrm{LDSA}}} - \frac{D_{\mathrm{LDSA}}}{2K_{\mathrm{LDSA}}}$$
$$= \left(1 - \frac{2}{\frac{1+z_0}{1-z_0}e^{bt} + 1}\right)\frac{b}{2K_{\mathrm{LDSA}}} - \frac{D_{\mathrm{LDSA}}}{2K_{\mathrm{LDSA}}}$$
$$= \frac{1}{2K_{\mathrm{LDSA}}}\left(b - \frac{2b}{\frac{1+z_0}{1-z_0}e^{bt} + 1} - D_{\mathrm{LDSA}}\right)$$
$$= \frac{1}{2K_{\mathrm{LDSA}}}\left(b - \frac{2}{\frac{2-1+z_0}{1-z_0}e^{bt} + 1} - D_{\mathrm{LDSA}}\right)$$
$$= \frac{1}{2K_{\mathrm{LDSA}}}\left(b - \frac{2b}{\left(\frac{2}{1-z_0} - 1\right)e^{bt} + 1} - D_{\mathrm{LDSA}}\right).$$

Substituting $z_0 = \frac{D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}}}{\sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}}} = \frac{D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}}}{b}$, it can be written

$$= \frac{1}{2K_{\mathrm{LDSA}}}\left(b - \frac{2b}{\left(\frac{2}{1 - \frac{D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}}}{b}} - 1\right)e^{bt} + 1} - D_{\mathrm{LDSA}}\right)$$
$$= \frac{1}{2K_{\mathrm{LDSA}}}\left(b - \frac{2b}{\left(\frac{2b}{b - D_{\mathrm{LDSA}} - 2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}}} - 1\right)e^{bt} + 1} - D_{\mathrm{LDSA}}\right)$$
$$= \frac{1}{2K_{\mathrm{LDSA}}}\left(b - D_{\mathrm{LDSA}} - 2b\left[\left(\frac{2b}{b - D_{\mathrm{LDSA}} - 2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}}} - 1\right)e^{bt} + 1\right]^{-1}\right).$$

As time approaches infinity, gradient of LDSA concentration approaches zero and LDSA concentration approaches background concentration $C_{\mathrm{LDSA,i,bg}}$ resulted from background source. Consequently, from the original differential equation, the background source S_{LDSA} can be formulated as

$$0 = -D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}} - K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}}^2 + S_{\mathrm{LDSA}}$$
$$\Leftrightarrow S_{\mathrm{LDSA}} = D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}} + K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}}^2.$$

It has to be noted that this step includes the assumption of constant dilution and coagulation coefficients for LDSA. Using the equation of background source, the auxiliary variable b can be expressed as

$$
\begin{aligned}
b &= \sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}}S_{\mathrm{LDSA}}} \\
&= \sqrt{D_{\mathrm{LDSA}}^2 + 4K_{\mathrm{LDSA}}\left(D_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}} + K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}}^2\right)} \\
&= \sqrt{D_{\mathrm{LDSA}}^2 + 4D_{\mathrm{LDSA}}K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}} + 4K_{\mathrm{LDSA}}{}^2C_{\mathrm{LDSA,i,bg}}^2} \\
&= \sqrt{(D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}})^2} \\
&= D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}}.
\end{aligned}
$$

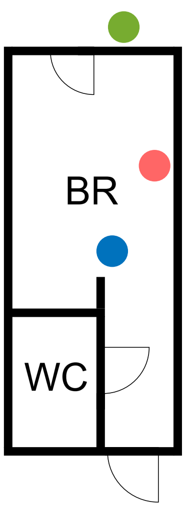
Finally, using this, LDSA-concentration can be written as

$$
\begin{aligned}
C_{\mathrm{LDSA,i}} &= \tfrac{1}{2K_{\mathrm{LDSA}}}\left(b - D_{\mathrm{LDSA}} - 2b\left[\left(\tfrac{2b}{b-D_{\mathrm{LDSA}}-2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}}} - 1\right)e^{bt} + 1\right]^{-1}\right) \\
&= \tfrac{1}{2K_{\mathrm{LDSA}}}\left(2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}} - 2(D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}})\left[\left(\tfrac{2(D_{\mathrm{LDSA}}+2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}})}{D_{\mathrm{LDSA}}+2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}}-D_{\mathrm{LDSA}}-2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,0}}} - 1\right)e^{bt} + 1\right]^{-1}\right) \\
&= \tfrac{1}{2K_{\mathrm{LDSA}}}\left(2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}} - 2(D_{\mathrm{LDSA}} + 2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}})\left[\left(\tfrac{2(D_{\mathrm{LDSA}}+2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}})}{2K_{\mathrm{LDSA}}(C_{\mathrm{LDSA,i,bg}}-C_{\mathrm{LDSA,i,0}})} - 1\right)e^{bt} + 1\right]^{-1}\right) \\
&= C_{\mathrm{LDSA,i,bg}} - \tfrac{D_{\mathrm{LDSA}}+2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}}}{K_{\mathrm{LDSA}}}\left[\left(\tfrac{D_{\mathrm{LDSA}}+2K_{\mathrm{LDSA}}C_{\mathrm{LDSA,i,bg}}}{K_{\mathrm{LDSA}}(C_{\mathrm{LDSA,i,bg}}-C_{\mathrm{LDSA,i,0}})} - 1\right)e^{bt} + 1\right]^{-1} \\
&= C_{\mathrm{LDSA,i,bg}} - \tfrac{b}{K_{\mathrm{LDSA}}}\left[\left(\tfrac{b}{K_{\mathrm{LDSA}}(C_{\mathrm{LDSA,i,bg}}-C_{\mathrm{LDSA,i,0}})} - 1\right)e^{bt} + 1\right]^{-1} \\
&= C_{\mathrm{LDSA,i,bg}} - b\left[\left(\tfrac{b}{C_{\mathrm{LDSA,i,0}}-C_{\mathrm{LDSA,i,bg}}} + K_{\mathrm{LDSA}}\right)e^{bt} - K_{\mathrm{LDSA}}\right]^{-1}.
\end{aligned}
$$

S2 Floor plans of the measurement sites

Table S2.1. Description of the acronyms used in floor plans.

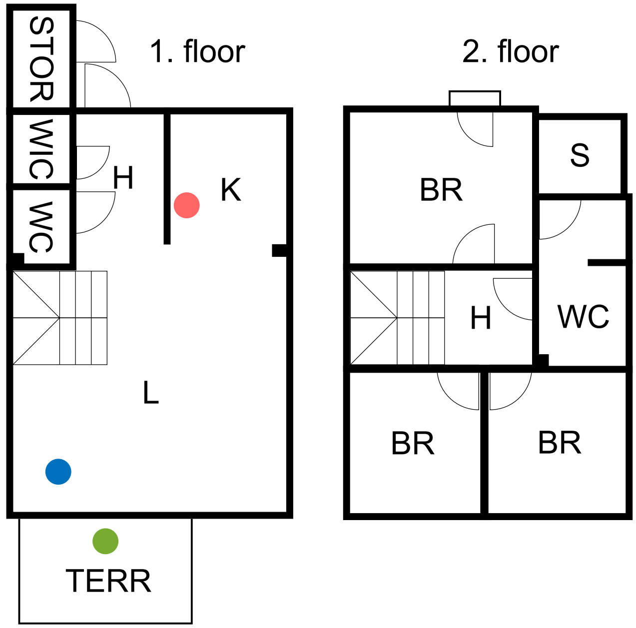
Acronym	Definition
BR	Bedroom
H	Hall
K	Kitchen
L	Living room
S	Sauna
STOR	Storage
TERR	Terrace
UTRM	Utility room
WC	Toilet/bathroom
WIC	Walk-in closet



CASE I	
Building type	Apartment
Floor area (m^2)	22
Area of cooking space (m^2)	16
Volume of cooking space (m^3)	~40
Ventilation system	Mechanical supply and exhaust
Extra ventilation during cooking	No
Location type	Urban

Stove
Living room sensor
Outdoor sensor

Figure S2.1. Floor plan and description of the apartment used in Case I.



CASE II	
Building type	Terraced house
Floor area (m^2)	85,0
Area of cooking space (m^2)	~45
Volume of cooking space (m^3)	~150
Ventilation system	Mechanical supply and exhaust
Extra ventilation during cooking	Yes
Location type	Suburban

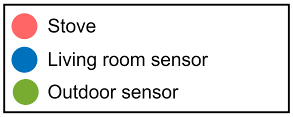
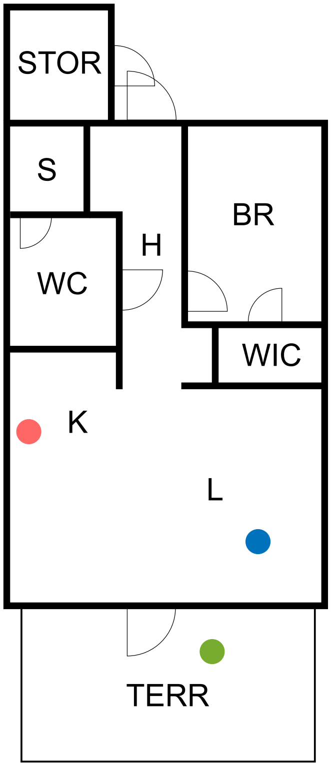


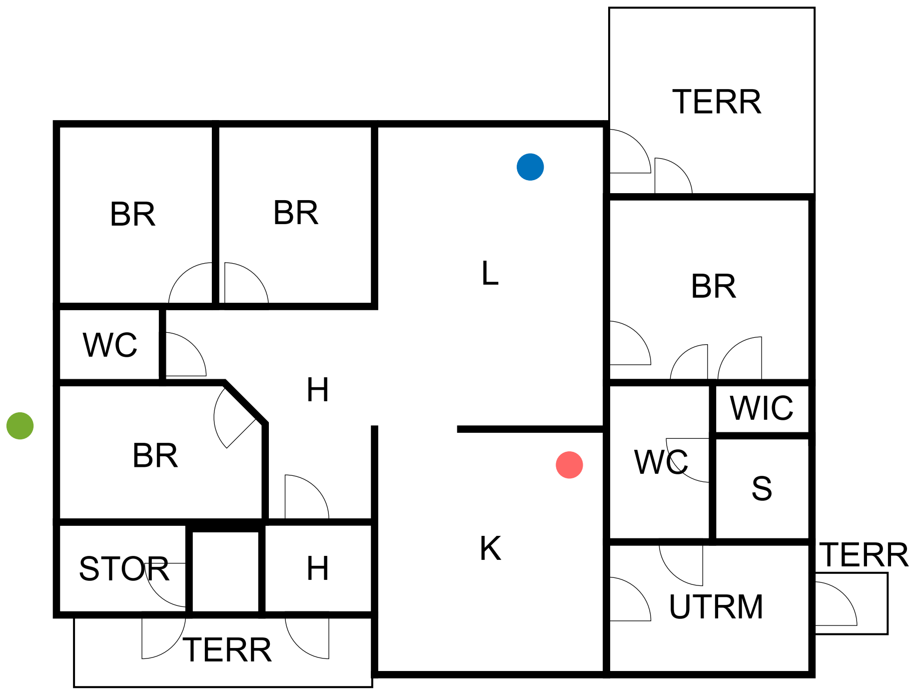
Figure S2.2. Floor plan and description of the apartment used in Case II



CASE III	
Building type	Terraced house
Floor area (m^2)	52.5
Area of cooking space (m^2)	~33
Volume of cooking space (m^3)	~86
Ventilation system	Mechanical supply and exhaust
Extra ventilation during cooking	Yes
Location type	Suburban

Stove
Living room sensor
Outdoor sensor

Figure S2.3. Floor plan and description of the apartment used in Case III



CASE IV	
Building type	Detached house
Floor area (m^2)	128,8
Area of cooking space (m^2)	57
Volume of cooking space (m^3)	~180
Ventilation system	Mechanical supply and exhaust
Extra ventilation during cooking	Yes
Location type	Suburban

Stove
Living room sensor
Outdoor sensor

Figure S2.4. Floor plan and description of the apartment used in Case IV.

S3 Fits of the decay function ignoring coagulation

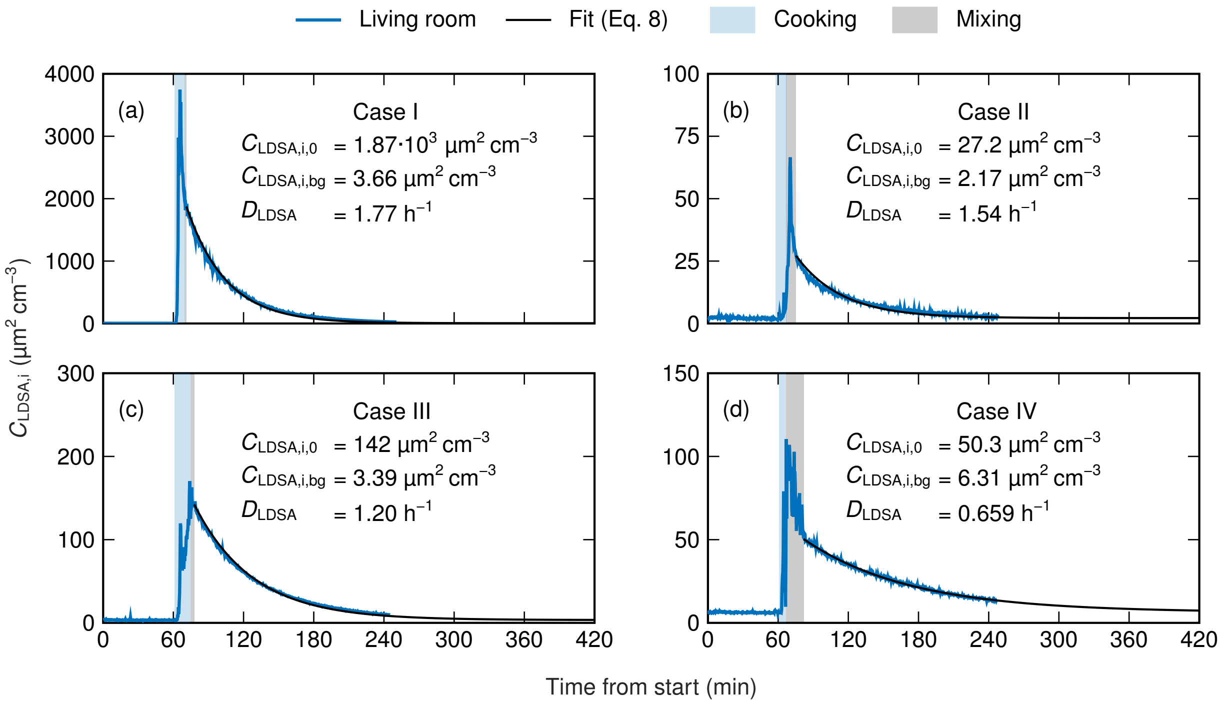


Figure S3.1. The fits and the fitting parameters of the decay function Eq. (8), not considering coagulation, in Cases I (a), II (b), III (c), and IV (d). Cooking events and mixing phases are highlighted.